Journal of Applied Mechanics and Technical Physics, Vol. 48, No. 2, pp. 260–264, 2007

NUMERICAL MODELING OF THE ROTATION OF A SOLID BODY WITH A LIQUID-FILLED CAVITY HAVING RADIAL RIBS

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UDC 531.3

The boundary-value problem of unsteady vortex flow of a viscous incompressible fluid in a cylindrical vessel with radial ribs rotating at a variable angular velocity is solved using a finite-difference method. The results of the solution are used to calculate the motion of a system of a solid body and a cavity filled with a liquid. The results are compared with available experimental data.

Key words: *viscous fluid, radial ribs, rotor flow, hydrodynamic loads.*

Introduction. The dynamic interaction of a rotating solid body with an incompressible fluid filling its cavity with radial ribs is studied in [1–3] and other papers. These studies deal primarily with applied problems of the dynamics of launchers and space vehicles equipped with liquid rocket engines and, hence, liquid fuel tanks.

The approach developed in [1–3] does not involve the solution of the boundary-value problem of the vortex motion of a fluid in a ribbed vessel for the calculation of hydrodynamic loads. The model proposed in the cited papers is phenomenological. In particular, the expression for the hydrodynamic load per unit length of a rib is similar to that proposed in [2] and is widely used in linear problems of the dynamics of solid bodies containing fluids in ribbed cavities:

$$f_n = \rho C \sqrt{u_n}\, U_n, \qquad U_n = \int\limits_{-\infty}^{t} \frac{\partial u_n}{\partial \tau} \, \frac{d\tau}{\sqrt{t-\tau}}.$$

Here $\boldsymbol{n}$ is the unit normal vector to the surface of the rib, $\boldsymbol{u}$ is the relative fluid velocity calculated under the assumption of no ribs, and C is an empirical function of the geometrical parameters of the ribs and their arrangement.

In the present paper, we consider a mathematical model that differs from the well-known models in that the hydrodynamic loads on the wetted surface are found by solving the corresponding boundary-value problem.

Formulation of the Problem. The solid body has a cavity with radial ribs filled with a viscous incompressible fluid. This system has a common axis of symmetry and rotates around this axis under the action of an external moment $M_x(t)$. The fluid flow is described by the Navier–Stokes and continuity equations. We consider only cylindrical cavities: a rectilinear circular cylinder or coaxial cylinders, i.e., a torus of rectangular cross section (Fig. 1). The dimensions of the cavity and the Reynolds number are assumed to be such that the bottom and cover of the cavity do not introduce noticeable perturbations to the fluid flow over the entire volume of the fluid. By virtue of this, the fluid flow can be considered two-dimensional. Under such assumptions, the equation of solid-body rotation is written as

$$(I^0 + I)\dot{\omega}_x + \rho \frac{d}{dt} \int\limits_{Q} r u_\theta \, dQ = M_x + M_s, \tag{1}$$

where $I^0 + I$ is the total moment of inertia of the solid body and the hardened fluid in the cavity, $\omega_x(t)$ is the angular velocity of the solid-body rotation, u_θ is the tangential component of the relative fluid velocity $\boldsymbol{u} = \{u_x \equiv 0, u_r, u_\theta\}$

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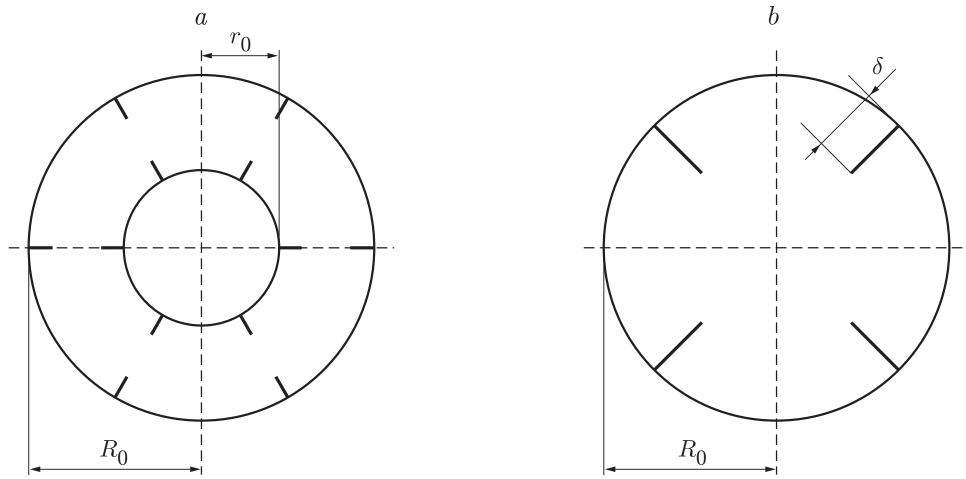

Fig. 1. Cross sections of cavities: (a) torus; (b) circular cylinder.

in the system of cylindrical coordinates $(0, x, r, \theta)$ attached to the solid, d/dt is the total time derivative in this coordinate system, Q is the region filled with the fluid, and M_s is the moment of the viscous friction forces on the cylindrical surfaces, bottom, and the cover of the cavity. The continuity and fluid-flow equations are written in the same reference system:

$$\operatorname{div}\boldsymbol{u} = 0,$$

$$\frac{\partial u_r}{\partial t} + \frac{1}{2}\frac{\partial u^2}{\partial r} - u_\theta(\mathrm{rot}_1\,\boldsymbol{u} + 2\omega_x) + \frac{1}{\rho}\frac{\partial p}{\partial r} - \nu\Big(\Delta u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\Big) = 0, \tag{2}$$

$$\frac{\partial\,(u_\theta + \omega_x r)}{\partial t} + \frac{1}{2}\frac{\partial u^2}{\partial r} + u_r(\mathrm{rot}_1\,\boldsymbol{u} + 2\omega_x) + \frac{1}{\rho}\frac{\partial p}{\partial \theta} - \nu\Big(\Delta u_\theta + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{u_\theta}{r^2}\Big) = 0.$$

Here $\mathrm{rot}_1\,\boldsymbol{u} = \boldsymbol{i}_1 \cdot \mathrm{rot}\,\boldsymbol{u}$ ($\boldsymbol{i}_1$ is the unit normal vector of the Ox axis).

Equations (1) and (2), the boundary attachment conditions on the wetted surface of the cavity and ribs, and the initial conditions constitute the coupled initial-boundary-value problem of solid-body rotation with a viscous fluid in the body cavity.

Computation Method. The coupled problem (1), (2) is solved using difference methods which approximate the differential equations with the first order of accuracy in time and space coordinates. In the computational scheme, the continuity equation in (2) is previously replaced by the Poisson equation for pressure:

$$\Delta p = -2\rho\Big(\Big(\frac{\partial u_r}{\partial r}\Big)^2 + \Big(\frac{\partial u_\theta}{\partial r}\Big)^2 - \Big(\omega_x + \frac{\partial u_\theta}{\partial r}\Big)\,\mathrm{rot}_1\,\boldsymbol{u} - \omega_x^2\Big) + \rho\,\frac{\omega_x}{r}\,\frac{\partial u_r}{\partial \theta}. \tag{3}$$

The boundary conditions for the Poisson equation are the derivatives with respect to pressure that are normal to the wetted surfaces, which follow from Eqs. (2) subject to the boundary attachment conditions for the velocity vector. The difference analogs of Eqs. (1)–(3) are solved using the Libman iterative procedure.

For a uniform distribution of the ribs over the cross-sectional circumference of the cavity, in view of the periodicity of the solution over the angle θ, Eqs. (2) are integrated over the range of the coordinate r from r_0 to R_0 and the range of the coordinate θ from 0 to $2\pi/m$, where m is the number of ribs (pairs of ribs).

The parameters of the computational scheme that ensure the stability and convergence of the numerical solution were determined in numerical experiments by varying the grid dimension over the space variables in the range of $2 \cdot 10^3$–$1.2 \cdot 10^5$ cells and the time integration step Δt in the range of 10^{-3}–10^{-5}. For the stability of the solution in fairly large time intervals and over a wide range of parameters $\omega_x(t)$ and $\dot{\omega}_x(t)$, the kinematic viscosity of the examined fluids was overestimated by a factor of 5–10 in order that the Reynolds number calculated on the rib width δ had an order of magnitude not larger than 10^3. The numerical experiment showed that this measure of ensuring stability had a weak effect (except in the initial period of time) on the sought-for integral characteristic of the problem — the hydrodynamic moment $M_h(t)$. We are not aware of an experimental support of this conclusion.

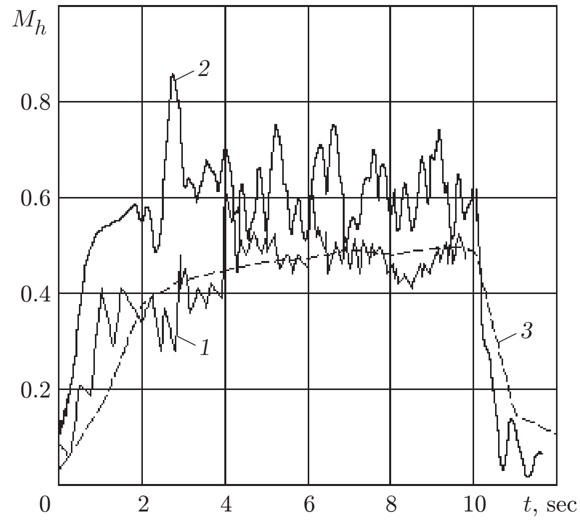

Fig. 2. Hydrodynamic moment versus time: 1) experimental curve; 2) calculated curve; 3) smoothed experimental curve.

In a study [4] of different experimental conditions — steady-state unbounded flow over a cylindrical obstacle, it is noted that an increase in the viscosity coefficient ν by approximately an order of magnitude (in the neighborhood of $\mathrm{Re} \approx 10^4$) shows little or no effect of viscosity on the Strouhal number $\mathrm{Sh} = f\delta/|u|$ (f is the frequency of stall of vortices) but has an important effect on the beginning of formation of a vortex system.

Because the experimental and calculated (with an artificially overestimated value of ν) curves of $M_h(t)$ presented in Fig. 2 have the largest difference during approximately the first 2 sec from the beginning of the deceleration of the cavity (i.e., on the segment of the formation of the vortex system), the above qualitative conclusions can be extended to the conditions of the problem being solved.

Calculation Results. Calculations were performed for the initial data of the problem corresponding to the physical experiments of [3, 5] in which the rotation of a solid body with a fluid was studied in two modes:

— in the spinup mode,

$$u_r = u_\theta = 0, \quad \omega_x(0) = \omega_x^0 = 0, \quad \dot{\omega}_x(0) = \dot{\omega}_x^0 > 0, \qquad t = 0,$$

$$\dot{\omega}_x(t) > 0, \qquad t > 0;$$

— in the deceleration mode,

$$u_r = u_\theta = 0, \quad \omega_x(0) = \omega_x^0 > 0, \quad \dot{\omega}_x(0) = \dot{\omega}_x^0 < 0, \qquad t = 0,$$

$$\dot{\omega}_x(t) < 0, \qquad t > 0.$$

Figure 3 gives experimental and calculated curves of $\omega_x(t)$ and $\dot{\omega}_x(t)$ for rotation of a solid body with a water-filled cylindrical cavity under the action of a constant external moment $M_x = 0.96$ N$\cdot$ m (the spinup mode). The cavity ($R_0 = 0.175$ m and height $H = 0.4$ m) has four equidistant ribs ($\delta = 0.034$ m). The moment of inertia is $I^0 + I = 1.03$ kg$\cdot$m^2 (here the moments of inertia of the dry solid and the rotating part of the experimental setup are taken into account).

The calculation for the deceleration mode was performed for a cavity shaped like a rectangular cross-section torus ($R_0 = 0.2$ m, $r_0 = 0.14$ m, and $H = 0.04$ m) with six pairs of equidistant ribs ($\delta = 0.02$ m). The fluid was wood alloy (eutectic) at a temperature of 128°C. The kinematic parameters $\omega_x(t)$ and $\dot{\omega}_x(t)$ were specified so (the initial velocities $\omega_x^0 = 9.87$ sec^{-1} and $\dot{\omega}_x = -0.983$ sec^{-2}) that complete stop of the solid-body rotation occurred in 10.04 sec.

Experimental and calculated curves of the hydrodynamic moment

$$M_h = -\rho \frac{d}{dt} \int\limits_Q r u_\theta \, dQ + M_s$$

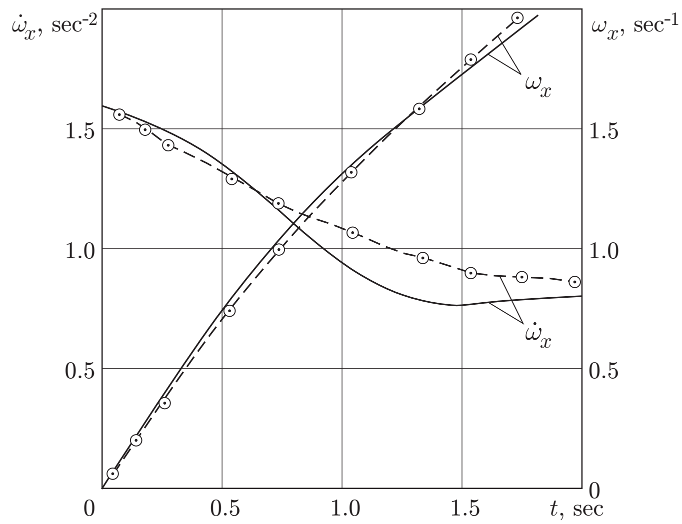

Fig. 3. Curves of $\omega_x(t)$ and $\dot{\omega}_x(t)$ for spinup of a cylindrical cavity by a moment of constant magnitude: the dashed curve refers to the experimental data of [3] and the solid curve refers to calculation results.

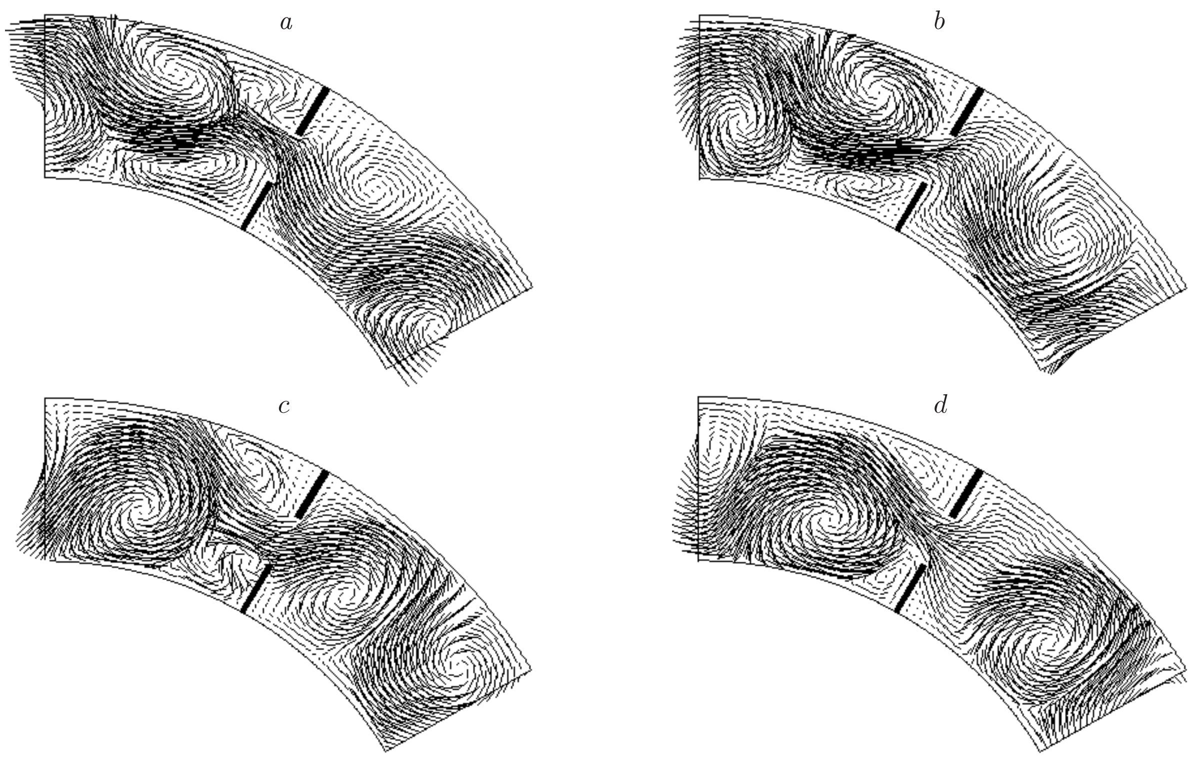

Fig. 4. Fluid velocity fields at various times: $t = 3$ (a), 4 (b), 5 (c), and 6 sec (d).

for these initial data are presented in Fig. 2. Under the assumption that the zigzag nature of the experimental curve is due to mechanical vibrations of the experimental facility (see [3]), this curve should be filtered from the indicated interferences. The result is a smooth curve 3, which is nearly monotonic on the segment $t = 0$–10 sec. Subsequently, the smoothed curve is used to determine the matching coefficients for identification of the mathematical model [5].

However, similarly to the experimental curve, the calculated curve of $M_h(t)$ has a zigzag (pulsating) nature. In this case, both the experimental and calculated curves exhibit maximum peaks (wide-scale pulsations) with an interval of approximately 1–2 sec and peaks of smaller amplitude (pulsations of smaller scales) at higher frequency. In the nomenclature of turbulence theory, these pulsations describe a deterministic chaos, or in our case, the irregular process of reconstruction of the vortex system of the fluid flow: the motion of the vortices and their evolution and interaction. Figure 4 gives fragments of a streak record of the relative-velocity field, which show the following reconstruction: alternation of the number of wide-scale vortices on the right and left of the ribs with a period of about 1 sec.

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